



$$A_{31} = (-1)^4 \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{\text{kof}} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -2 & 2 & 1 \end{bmatrix}$$

provera:

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & -1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & -2 \\ -1 & -1 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

inverzna matrica matrice A

2) Nadi inverznu matricu matrice  $B = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}$ .

Rj:  $B^{-1} = \frac{1}{\det B} B_{\text{kof}}^T$ ,  $\det B = \begin{vmatrix} 3 & 2 & 4 \\ 2 & 3 & 4 \\ 1 & 1 & 2 \end{vmatrix} \begin{vmatrix} 1 & -1 & -2 \\ 1 & -1 & -2 \\ 1 & -1 & -2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} -1 & -2 \\ 1 & 0 \end{vmatrix} = 2$

$$B_{11} = (-1)^2 \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 2$$

$$B_{21} = (-1)^2 \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0$$

$$B_{31} = (-1)^4 \begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix} = -4$$

$$B_{12} = (-1)^3 \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0$$

$$B_{22} = (-1)^4 \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 2$$

$$B_{32} = (-1)^5 \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = -4$$

$$B_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -1$$

$$B_{23} = (-1)^5 \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -1$$

$$B_{33} = (-1)^6 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 5$$

$$B_{\text{kof}}^T = \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -4 \\ -1 & -1 & 5 \end{bmatrix}$$

$$B^{-1} = \frac{1}{2} \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -4 \\ -1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{5}{2} \end{bmatrix} \text{ tražena inverzna matrica}$$

3) Nadi inverznu matricu matrice  $C = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$ .

Rj:  $C^{-1} = \frac{1}{\det C} C_{\text{kof}}^T$ ,  $\det C = \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} = 3$

$$C_{11} = (-1)^2 \cdot 4 = 4$$

$$C_{21} = (-1)^3 \cdot 1 = -1$$

$$C_{\text{kof}}^T = \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} \frac{4}{3} & -\frac{1}{3} \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$$

$$C_{12} = (-1)^3 \cdot 5 = -5$$

$$C_{22} = (-1)^4 \cdot 2 = 2$$

4) Nadi inverznu matricu sledećih matrica:

a)  $A = \begin{bmatrix} 3 & 4 & 4 \\ 1 & 6 & 1 \\ 2 & 3 & 3 \end{bmatrix}$

c)  $C = \begin{bmatrix} 7 & 3 & 3 \\ 6 & 3 & 4 \\ -1 & -2 & -3 \end{bmatrix}$

b)  $B = \begin{bmatrix} -3 & -1 & -1 \\ 1 & 3 & 2 \\ -2 & -1 & -2 \end{bmatrix}$

Rješenja:

a)  $A^{-1} = \begin{bmatrix} \frac{3}{5} & 0 & -4 \\ -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{14}{5} \end{bmatrix}$

b)  $B^{-1} = \begin{bmatrix} -\frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ -\frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$

c)  $\det C = 8$

# Matricne jednačine

U sljedećim primjerima neka su  $A, B, C, X$  neke date kvadratne matrice.

$$A^{-1} \cdot B \neq B \cdot A^{-1}$$

$$A \cdot B \neq B \cdot A$$

Matrice se ne mogu dijeliti.

Da bismo odredili nepoznatu  $X$  u matricnoj jednačini; <sup>prvo</sup> trebamo izvesti formulu za nepoznatu  $X$ .

⊕  $A \cdot X = B$  /  $A^{-1}$  sa lijeve strane

$$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$$

$$1 \cdot X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B$$

⊕  $X^{-1} \cdot A = B^{-1}$  /  $A^{-1}$  sa desne strane

$$X^{-1} \cdot A \cdot A^{-1} = B^{-1} \cdot A^{-1}$$

$$X^{-1} \cdot 1 = B^{-1} \cdot A^{-1}$$

$$X^{-1} = B^{-1} \cdot A^{-1} \quad | \cdot (-1)$$

$$X = A \cdot B$$

⊕  $A \cdot X \cdot B = C$  /  $A^{-1}$  sa lijeve strane

$$A^{-1} \cdot A \cdot X \cdot B = A^{-1} \cdot C$$

$$1 \cdot X \cdot B = A^{-1} \cdot C \quad | B^{-1} \text{ sa desne strane}$$

$$X \cdot B \cdot B^{-1} = A^{-1} \cdot C \cdot B^{-1}$$

$$X \cdot 1 = A^{-1} \cdot C \cdot B^{-1}$$

$$X = A^{-1} \cdot C \cdot B^{-1}$$

⊕  $A^{-1} \cdot X = X - 1$

$$A^{-1} \cdot X - X = -1$$

$$\underbrace{(A^{-1} - 1)}_B \cdot X = -1$$

$$B \cdot X = -1 \quad | \cdot B^{-1} \text{ sa lijeve strane}$$

$$B^{-1} \cdot B \cdot X = -B^{-1} \cdot 1$$

$$X = -B^{-1}$$

$$X = -(A^{-1} - 1)^{-1}$$

⊕  $A \cdot X + 1 = X - 21$

$$A \cdot X - X = -1 - 21$$

$$\underbrace{(A - 1)}_B \cdot X = -31$$

$$B \cdot X = -31 \quad | \cdot B^{-1} \text{ sa desne strane}$$

$$B^{-1} \cdot B \cdot X = B^{-1} \cdot (-31)$$

$$1 \cdot X = -31 B^{-1}$$

$$X = -31 (A - 1)^{-1}$$

$$\textcircled{\#} \underbrace{(A+3I)}_C (X-1) = B$$

$C(X-1) = B$   $\cdot C^{-1}$  sa lijeve strane

$$C^{-1}C(X-1) = C^{-1} \cdot B$$

$$X-1 = C^{-1} \cdot B$$

$$X = C^{-1} \cdot B + 1$$

$$X = (A+3I)^{-1} \cdot B + 1$$

$$\textcircled{\#} B^{-1} \cdot X \cdot A = (3B+2I)^{-1}$$

$\cdot B$  sa lijeve strane

$$B \cdot B^{-1} \cdot X \cdot A = B(3B+2I)^{-1}$$

$$X \cdot A = B(3B+2I)^{-1} \quad \cdot A^{-1} \text{ sa desne strane}$$

$$X = B(3B+2I)^{-1} \cdot A^{-1}$$

$$\textcircled{\#} (AXB)^{-1} = B^{-1}(X^{-1}+B)$$

$\cdot (AXB)$  sa lijeve strane

$$(AXB)(AXB)^{-1} = AX \underbrace{B B^{-1}}_I (X^{-1}+B)$$

$$I = AX(X^{-1}+B)$$

$$I = AX X^{-1} + AXB$$

$$I = A + AXB$$

$$AXB = I - A \quad \cdot A^{-1} \text{ sa lijeve str.}$$

$$A^{-1}AXB \cdot B^{-1} = A^{-1}(I-A) \cdot B^{-1}$$

$$X = A^{-1}(I-A) \cdot B^{-1}$$

$\textcircled{10}$  Riješiti matricnu jednačinu

$$X \cdot \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Rj:  $X \cdot A = B$   $\cdot A^{-1}$  sa desne str.

$$X = B \cdot A^{-1}, \quad A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 2 & 3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot A_{kof}^T$$

$$A_{11} = (-1)^2 \begin{vmatrix} -2 & -3 \\ 3 & 5 \end{vmatrix} = -1$$

$$A_{12} = (-1)^3 \begin{vmatrix} -1 & -3 \\ 2 & 5 \end{vmatrix} = -1$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & -2 \\ 2 & 3 \end{vmatrix} = 1$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & 1 \\ 3 & 5 \end{vmatrix} = 2$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 3$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1$$

$$\det A = \begin{vmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 2 & 3 & 5 \end{vmatrix} \begin{vmatrix} I_k - III_k \\ II_k - III_k \end{vmatrix} \begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & -3 \\ -3 & -2 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -3 & -2 \end{vmatrix} = -1$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} = -1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ -1 & -3 \end{vmatrix} = 2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = -1$$

$$A_{kof}^T = \begin{bmatrix} -1 & -2 & -1 \\ -1 & 3 & 2 \\ 1 & -1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -3 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$X = B \cdot A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 1 & -3 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 \\ 1 & 2 & 1 \\ 1 & -1 & -1 \end{bmatrix} \quad \text{rješuje matricne jednačine}$$

2) Riješiti matricnu jednačinu  $A \cdot X = X + I$

ako je  $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 1 & -1 \end{bmatrix}$ .

Rj.  $A \cdot X = X + I$   $C = A - I = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & -2 \\ 3 & 1 & -2 \end{bmatrix}$

$$A \cdot X - X = I$$

$$(A - I) \cdot X = I \quad | \cdot (A - I)^{-1} \text{ sa lijeve strane}$$

$$C^{-1} = \frac{1}{\det C} C_{\text{kof}}^T$$

$$(A - I)(A - I)^{-1} X = (A - I)^{-1} \cdot I$$

$$X = (A - I)^{-1} \cdot I$$

$$\det C = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 2 & -2 \\ 3 & 1 & -2 \end{vmatrix} \xrightarrow{\text{II}_k + \text{III}_k} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & -2 \\ 3 & -1 & -2 \end{vmatrix}$$

$$C_{11} = (-1)^2 \begin{vmatrix} 2 & -2 \\ 1 & -2 \end{vmatrix} = -2$$

$$C_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = -1$$

$$C_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 2 & -2 \end{vmatrix} = 0$$

$$C_{12} = (-1)^3 \begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix} = -4$$

$$C_{22} = (-1)^4 \begin{vmatrix} 0 & 1 \\ 3 & -2 \end{vmatrix} = -3$$

$$C_{32} = (-1)^5 \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 1$$

$$C_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$$C_{23} = (-1)^5 \begin{vmatrix} 0 & -1 \\ 3 & 1 \end{vmatrix} = -3$$

$$C_{33} = (-1)^6 \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1$$

$$C_{\text{kof}}^T = \begin{bmatrix} -2 & -1 & 0 \\ -4 & -3 & 1 \\ -5 & -3 & 1 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & -1 \\ 5 & 3 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & -1 \\ 5 & 3 & -1 \end{bmatrix} \quad \text{rjesenje}$$

3) Riješiti matricnu jednačinu  $(A+B)^{-1} A \cdot X^{-1} = B^{-1}$  gdje su matrice  $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$  i  $B = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$ .

Rj.  $(A+B)^{-1} A \cdot X^{-1} = B^{-1}$   $| \cdot (A+B) \text{ sa lijeve strane}$

$$X^{-1} = A^{-1} (A+B) \cdot B^{-1} \quad | \cdot A$$

$$(A+B)(A+B)^{-1} A \cdot X^{-1} = (A+B) \cdot B^{-1}$$

$$X = B(A+B)^{-1} A$$

$$A \cdot X^{-1} = (A+B) \cdot B^{-1} \quad | \cdot A^{-1} \text{ sa lijeve strane}$$

$$A^{-1} \cdot A \cdot X^{-1} = A^{-1} (A+B) \cdot B^{-1}$$

$$C = A+B = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}, \quad C^{-1} = \frac{1}{\det C} C_{\text{kof}}^T, \quad \det C = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix} = 13,$$

$$C_{11} = (-1)^2 \cdot 3 = 3$$

$$C_{12} = (-1)^3 \cdot 1 = -1$$

$$C_{21} = (-1)^2 \cdot (-1) = 1$$

$$C_{22} = (-1)^4 \cdot 4 = 4$$

$$C_{\text{kof}}^T = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} \frac{3}{13} & \frac{1}{13} \\ -\frac{1}{13} & \frac{4}{13} \end{bmatrix}$$

$$X - B \cdot C^{-1} \cdot A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \cdot \frac{1}{13} \cdot \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 7 & 5 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 15 & -6 \\ 9 & 12 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{15}{13} & -\frac{6}{13} \\ \frac{9}{13} & \frac{12}{13} \end{bmatrix} \text{ rješenje matricne jednačine}$$

4. Riješiti matricnu jednačinu  $(A+3I)(X-I) = B$ , ako je

$$A = \begin{bmatrix} -2 & 5 & -2 \\ 2 & 8 & 0 \\ -1 & -5 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 21 & 1 \\ 2 & 50 & -2 \\ 1 & -22 & 0 \end{bmatrix} \quad ; \quad I \text{ jedinična matrica.}$$

Rj.  $(A+3I)(X-I) = B \quad / \cdot (A+3I)^{-1}$  sa lijeve strane  $C^{-1} = \frac{1}{\det C} C_{\text{kof}}^T$

$$(A+3I)^{-1}(A+3I)(X-I) = (A+3I)^{-1} \cdot B$$

$$X-I = (A+3I)^{-1} \cdot B$$

$$X = (A+3I)^{-1} \cdot B + I$$

$$\det C = \begin{vmatrix} 1 & 5 & -2 \\ 2 & 11 & 0 \\ -1 & -5 & 1 \end{vmatrix} \xrightarrow{I_2 + III_2}$$

$$= \begin{vmatrix} 0 & 0 & -1 \\ 2 & 11 & 0 \\ -1 & -5 & 1 \end{vmatrix} = (-1) \begin{vmatrix} 2 & 11 \\ -1 & -5 \end{vmatrix} = -1$$

$$C = A+3I = \begin{bmatrix} -2 & 5 & -2 \\ 2 & 8 & 0 \\ -1 & -5 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 5 & -2 \\ 2 & 11 & 0 \\ -1 & -5 & 1 \end{bmatrix}$$

$$C_{11} = (-1)^2 \begin{vmatrix} 11 & 0 \\ -5 & 1 \end{vmatrix} = 11$$

$$C_{12} = (-1)^3 \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} = -2$$

$$C_{13} = (-1)^4 \begin{vmatrix} 2 & 11 \\ -1 & -5 \end{vmatrix} = 1$$

$$C_{21} = (-1)^3 \begin{vmatrix} 5 & -2 \\ -5 & 1 \end{vmatrix} = 5$$

$$C_{31} = (-1)^4 \begin{vmatrix} 5 & -2 \\ 11 & 0 \end{vmatrix} = 22$$

$$C_{22} = (-1)^4 \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} = -1$$

$$C_{32} = (-1)^5 \begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix} = -4$$

$$C_{23} = (-1)^5 \begin{vmatrix} 1 & 5 \\ -1 & -5 \end{vmatrix} = 0$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & 5 \\ 2 & 11 \end{vmatrix} = 1$$

$$C_{\text{kof}}^T = \begin{bmatrix} 11 & 5 & 22 \\ -2 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} -11 & -5 & -22 \\ 2 & 1 & 4 \\ -1 & 0 & -1 \end{bmatrix}$$

$$C^{-1} \cdot B = \begin{bmatrix} -11 & -5 & -22 \\ 2 & 1 & 4 \\ -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 21 & 1 \\ 2 & 50 & -2 \\ 1 & -22 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 4 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

$$X = (A+3I)^{-1} \cdot B + I = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 4 & 0 \\ 2 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 5 & 0 \\ 2 & 1 & 0 \end{bmatrix} \text{ riješi matricne jednačine}$$

5) Riješiti matricnu jednačinu  $(X^{-1} + B^{-1})^{-1} = AX$  ako su

$$A = \begin{bmatrix} 3 & 3 & 2 \\ -4 & 1 & -4 \\ -3 & 1 & -3 \end{bmatrix} \text{ i } B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix}.$$

Rj.  $(X^{-1} + B^{-1})^{-1} = AX$  /  $(X^{-1} + B^{-1})$  sa  
desne strane

$$A^{-1} = \frac{1}{\det A} \cdot A_{\text{kof}}^{-1}$$

$$(X^{-1} + B^{-1})^{-1} \cdot (X^{-1} + B^{-1}) = AX(X^{-1} + B^{-1})$$

$$\det A = \begin{vmatrix} 3 & 3 & 2 \\ -4 & 1 & -4 \\ -3 & 1 & -3 \end{vmatrix} \begin{matrix} |k-III| \\ |k-II| \end{matrix}$$

$$= \begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -4 \\ 1 & -3 \end{vmatrix}$$

$$= -3 + 4 = 1$$

$$I = A + AXB^{-1}$$

$$AXB^{-1} = I - A \text{ / } \cdot A^{-1} \text{ sa lijeve str.}$$

$$\cdot B \text{ sa desne str.}$$

$$A^{-1} \cdot A \cdot X \cdot B^{-1} \cdot B = A^{-1}(I - A) \cdot B$$

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & -4 \\ 1 & -3 \end{vmatrix} = 1$$

$$X = A^{-1}(I - A) \cdot B$$

$$A_{12} = (-1)^3 \begin{vmatrix} -4 & -4 \\ -3 & -3 \end{vmatrix} = 0$$

$$A_{21} = (-1)^3 \begin{vmatrix} 3 & 2 \\ 1 & -3 \end{vmatrix} = 11$$

$$A_{31} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 1 & -4 \end{vmatrix} = -14$$

$$A_{13} = (-1)^4 \begin{vmatrix} -4 & 1 \\ -3 & 1 \end{vmatrix} = -1$$

$$A_{22} = (-1)^4 \begin{vmatrix} 3 & 2 \\ -3 & -3 \end{vmatrix} = -3$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 2 \\ -4 & -4 \end{vmatrix} = 4$$

$$A_{\text{kof}} = \begin{bmatrix} 1 & 0 & -1 \\ 11 & -3 & -12 \\ -14 & 4 & 15 \end{bmatrix}$$

$$A_{23} = (-1)^5 \begin{vmatrix} 3 & 3 \\ -3 & 1 \end{vmatrix} = -12$$

$$A_{33} = (-1)^6 \begin{vmatrix} 3 & 3 \\ -4 & 1 \end{vmatrix} = 15$$

$$A^{-1} = \begin{bmatrix} 1 & 11 & -14 \\ 0 & -3 & 4 \\ -1 & -12 & 15 \end{bmatrix}$$

$$X = A^{-1}(I - A) \cdot B = \begin{bmatrix} 1 & 11 & -14 \\ 0 & -3 & 4 \\ -1 & -12 & 15 \end{bmatrix} \begin{bmatrix} -2 & -3 & -2 \\ 4 & 0 & 4 \\ 3 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 11 & -14 \\ 0 & -3 & 4 \\ -1 & -12 & 15 \end{bmatrix} \begin{bmatrix} -5 & 8 & -6 \\ 4 & -4 & 12 \\ 2 & -2 & 10 \end{bmatrix} = \begin{bmatrix} 11 & -8 & -14 \\ -4 & 4 & 4 \\ -13 & 10 & 12 \end{bmatrix} \text{ riješi matricne jednačine}$$

6) Riješiti matricnu jednačinu:

$$X \cdot \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix} = X \cdot \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

Ako označimo  $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{bmatrix}$  i  $B = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix}$  imamo

$$XA + B = XB$$

$$XA - XB = -B$$

$$X(A-B) = -B \quad / \cdot (A-B)^{-1} \text{ s obje strane}$$

$$X = -B(A-B)^{-1}$$

$$C = A - B = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$C_{11} = (-1)^2 \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = -1$$

$$C_{21} = 1 \quad C_{31} = 2$$

$$C_{22} = 1 \quad C_{32} = 0$$

$$C_{23} = 0 \quad C_{33} = 2$$

$$C^{-1} = \frac{1}{\det C} \cdot C^T$$

$$C_{12} = (-1)^3 \cdot 1 = -1$$

$$C_{13} = -2$$

$$\det C = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix} \xrightarrow{I_k + II_k} \begin{vmatrix} 1 & -1 & -1 \\ 2 & 1 & -1 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= (-2) \begin{vmatrix} -1 & -1 \\ -1 & 0 \end{vmatrix} = (-2) \cdot (-1) = 2$$

$$C^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix}$$

$$X = -B \cdot C^{-1} = - \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \frac{1}{2} \begin{bmatrix} -1 & 1 & 2 \\ -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix}$$

$$X = -\frac{1}{2} \begin{bmatrix} -15 & 5 & 12 \\ -8 & 1 & 8 \\ -11 & 7 & 10 \end{bmatrix} = \begin{bmatrix} \frac{15}{2} & -\frac{5}{2} & -6 \\ \frac{4}{2} & -\frac{1}{2} & -4 \\ \frac{11}{2} & -\frac{7}{2} & -5 \end{bmatrix} \text{ rješenje matricne jednačine}$$

7. Riješiti matricnu jednačinu  $(A+X)(B-2I) = A$ , ako su

$$A = \begin{bmatrix} -2 & -3 & -4 \\ 1 & -2 & 3 \\ 4 & 3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -2 & 1 \\ 1 & -1 & 0 \\ 1 & -2 & 2 \end{bmatrix}, \quad I \text{ jedinična matrica.}$$

8. Riješiti matricnu jednačinu  $A^{-1}X + B = AX$ , ako su

$$A = \begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}; \quad B = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}.$$

9. Riješiti matricnu jednačinu  $(XB^{-1})^{-1} = X^{-1} + A$ , ako su

$$A = \begin{bmatrix} -1 & 3 & 1 \\ 1 & 2 & 3 \\ 0 & 3 & 2 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}.$$

Rješenja:

$$7. \quad X = \begin{bmatrix} -2 & 10 & -1 \\ 2 & 2 & -5 \\ -6 & -14 & 19 \end{bmatrix}$$

$$8. \quad X = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$$

$$9. \quad X = \begin{bmatrix} 3 & -\frac{3}{2} & -\frac{17}{2} \\ 1 & -1 & -5 \\ 0 & \frac{5}{2} & \frac{15}{2} \end{bmatrix}$$



# Data je matricna jednačina  $A(X-B)^{-1} = B^{-1}A$  i matrice

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} ; B = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix}.$$

a) Koji uslov moraju zadovoljavati matrice A i B da bi data jednačina imala rješenje  $X=2B$ ?

b) Riješiti data jednačinu ako matrice A i B ne zadovoljavaju uslov dobijen pod a)

Rj. a)  $A(X-B)^{-1} = B^{-1}A$

$$X = 2B$$

$A \cdot B^{-1} = B^{-1}A$  uslov koji moraju zadovoljavati matrice A i B da bi data jednačina imala rješenje  $X=2B$ .

Ustav možemo pisati i na drugi način:

$$A = B^{-1}AB$$

ili

$$B = A^{-1} \cdot B \cdot A$$

b)  $A(X-B)^{-1} = B^{-1}A$   $\cdot (X-B)$  sa desne str

$$B^{-1}A(X-B) = A \quad \cdot B \text{ sa lijeve str.}$$

$$A(X-B) = BA \quad \cdot A^{-1} \text{ sa lijeve str.}$$

$$X-B = A^{-1}BA$$

$$X = A^{-1}BA + B$$

i odatudje možemo pročitati uslov koji smo dobili pod a) (ako je  $B = A^{-1}BA$  tada jednačina ima rješenje  $X=2B$ )

Proverimo da li je

$$B = A^{-1}BA.$$

Nadamo prvo  $A^{-1}$

$$A^{-1} = \frac{1}{\det A} \cdot A_{\text{kof}}^T$$

$$\det A = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$$

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$$

$$A_{21} = (-1)^2 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -2$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$A_{\text{kof}} = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A_{12} = (-1)^3 \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1$$

$$A_{13} = (-1)^4 \begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} = 0$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = 1$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_{\text{kof}}^T = \begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = 2 \begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}, \quad B \cdot A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 3 & 0 & 4 \end{bmatrix}$$

$$A^{-1} \cdot B \cdot A = 2 \begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 3 & 0 & 4 \end{bmatrix} = 2 \begin{bmatrix} -2 & -2 & -2 \\ -1 & 3 & -1 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} \\ \frac{5}{2} & \frac{3}{2} & \frac{7}{2} \end{bmatrix}$$

ovdje vidimo da matrice A i B ne zadovoljavaju uslov dobijen pod a)

$$X = A^{-1}BA + B = \begin{bmatrix} -1 & -1 & -1 \\ -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} \\ \frac{5}{2} & \frac{3}{2} & \frac{7}{2} \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \\ \frac{11}{2} & \frac{1}{2} & \frac{11}{2} \end{bmatrix} \text{ rješenje matricne jednačine}$$

#) Riješiti matricnu jednačinu  $X \cdot A^{-1} = B^{-1}$  ako su

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -4 \end{bmatrix}; \quad B = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}.$$

Rj:  $X \cdot A^{-1} = B^{-1}$  / A sa desne strane

$$\det B = \begin{vmatrix} 2 & 1 & -1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{vmatrix} \xrightarrow{|R_2 - R_1|}$$

$$\underbrace{X A^{-1} \cdot A}_{I} = B^{-1} \cdot A$$

$$X = B^{-1} \cdot A$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = 0 + 1$$

$$\det B = 1$$

$$B^{-1} = \frac{1}{\det B} B_{kof}^T$$

$$B_{11} = (-1)^2 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1$$

$$B_{21} = (-1)^3 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = -1$$

$$B_{31} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = -1$$

$$B_{12} = (-1)^3 \begin{vmatrix} 2 & -2 \\ -1 & 1 \end{vmatrix} = 0$$

$$B_{22} = (-1)^4 \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 1$$

$$B_{32} = (-1)^5 \begin{vmatrix} 2 & -1 \\ 2 & -2 \end{vmatrix} = 2$$

$$B_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = 1$$

$$B_{23} = (-1)^5 \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = -1$$

$$B_{33} = (-1)^6 \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 0$$

$$B_{kof} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix},$$

$$B_{kof}^T = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix},$$

$$B^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

$$X = B^{-1} \cdot A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -4 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 7 \\ 5 & 4 & -7 \\ -1 & 1 & 3 \end{bmatrix}$$

$$2 - 3 - 1$$

$$0 + 3 + 2$$

$$2 - 3 + 0$$

$$3 - 2 - 1$$

$$0 + 2 + 2$$

$$3 - 2 + 0$$

$$4 - 1 + 4$$

$$0 + 1 - 8$$

$$4 - 1 + 0$$

$$X = \begin{bmatrix} -2 & 0 & 7 \\ 5 & 4 & -7 \\ -1 & 1 & 3 \end{bmatrix}$$

traženo rješenje.

# Riješiti matricnu jednačinu  $X^{-1}AB = B^{-1}A^{-1}$ ,

$$A = \begin{bmatrix} 1 & 1 & 6 \\ 2 & -3 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -4 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$

Rj:  $X^{-1}AB = B^{-1}A^{-1}$

$$X^{-1}AB = (AB)^{-1} \quad / (AB)^{-1} \text{ sa desne strane}$$

$$X^{-1} = (AB)^{-1} (AB)^{-1}$$

$$X = (AB) \cdot (AB)$$

$$X = (AB)^2$$

$$A \cdot B = \begin{bmatrix} 1 & 1 & 6 \\ 2 & -3 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 1 & -4 & 1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 9 & -5 & 13 \\ 1 & 10 & -3 \\ 2 & -4 & 3 \end{bmatrix}$$

$$\begin{array}{ccc} 2+1+6 & 4-3+0 & 0+1+1 \\ -1-4+0 & -2+12+0 & 0-4+0 \\ 0+1+12 & 0-3+0 & 0+1+2 \end{array}$$

$$(AB)^2 = \begin{bmatrix} 9 & -5 & 13 \\ 1 & 10 & -3 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 9 & -5 & 13 \\ 1 & 10 & -3 \\ 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 102 & -147 & 171 \\ 12 & 107 & -26 \\ 20 & -62 & 47 \end{bmatrix}$$

$$\begin{array}{ccc} 81-5+26 & -45-50-52 & 117+15+39 \\ 9+10-6 & -5+100+12 & 12-30-9 \\ 18-4+6 & -10-40-12 & 26+12+9 \end{array}$$

$$X = \begin{bmatrix} 102 & -147 & 171 \\ 12 & 107 & -26 \\ 20 & -62 & 47 \end{bmatrix}$$

Ⓝ Riješiti matricnu jednačinu  $(A+I)^{-1} \cdot X \cdot (3A+I) = 2A$   
 gdje je  $I$  jedinična matrica drugog reda a

$$A = \begin{bmatrix} 7 & 8 \\ -6 & -7 \end{bmatrix}$$

Rj:  $(A+I)^{-1} \cdot X \cdot (3A+I) = 2A$  /  $(A+I)$  sa lijeve strane

$$X \cdot (3A+I) = (A+I) \cdot 2A$$
 /  $(3A+I)^{-1}$  sa desne strane

$$X = (A+I) \cdot 2A \cdot (3A+I)^{-1}$$

$$A = \begin{bmatrix} 7 & 8 \\ -6 & -7 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad A+I = \begin{bmatrix} 8 & 8 \\ -6 & -6 \end{bmatrix}$$

$$\frac{20 \cdot 22}{40} = \frac{40}{440}$$

$$3A+I = \begin{bmatrix} 22 & 24 \\ -18 & -20 \end{bmatrix} \quad 3A = \begin{bmatrix} 21 & 24 \\ -18 & -21 \end{bmatrix}$$

$$\frac{18 \cdot 24}{72} = \frac{36}{432}$$

Označimo sa  $B = 3A - I$  pa pronadimo  $B^{-1}$

$$B^{-1} = \frac{1}{\det B} \cdot B_{kof}^T \quad \det B = \begin{vmatrix} 22 & 24 \\ -18 & -20 \end{vmatrix} = -440 + 432 = -8$$

$$B_{11} = (-1)^2 \cdot (-20) = -20$$

$$B_{21} = (-1)^3 \cdot 24 = -24$$

$$B_{kof} = \begin{bmatrix} -20 & 18 \\ -24 & 22 \end{bmatrix}$$

$$B_{12} = (-1)^3 \cdot (-18) = 18$$

$$B_{22} = (-1)^4 \cdot 22 = 22$$

$$B^{-1} = \frac{-1}{8} \begin{bmatrix} -20 & -24 \\ 18 & 22 \end{bmatrix} = (3A+I)^{-1}$$

$$X = (A+I) \cdot 2A \cdot (3A+I)^{-1} = \begin{bmatrix} 8 & 8 \\ -6 & -6 \end{bmatrix} \cdot 2 \cdot \begin{bmatrix} 7 & 8 \\ -6 & -7 \end{bmatrix} \cdot \frac{-1}{8} \begin{bmatrix} -20 & -24 \\ 18 & 22 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 4 & 4 \\ -3 & -3 \end{bmatrix} \cdot 2 \begin{bmatrix} 7 & 8 \\ -6 & -7 \end{bmatrix} \cdot \frac{-1}{8} \cdot 2 \begin{bmatrix} -10 & -12 \\ 9 & 11 \end{bmatrix} = 8 \cdot \frac{-1}{8} \begin{bmatrix} 4 & 4 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ -6 & -7 \end{bmatrix} \begin{bmatrix} -10 & -12 \\ 9 & 11 \end{bmatrix}$$

$$= (-1) \begin{bmatrix} 4 & 4 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} -10 & -12 \\ 9 & 11 \end{bmatrix} = (-1) \begin{bmatrix} -4 & -4 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -3 & -3 \end{bmatrix}$$

rečnije matricne jednačine

#) Riješiti matricnu jednačinu  $(AX+B)^{-1} = B^{-1}(X^{-1}+B)$

ako je  $A = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ .

R.)  $(AX+B)^{-1} = B^{-1}(X^{-1}+B)$

$B^{-1}X^{-1}A^{-1} = B^{-1}X^{-1} + B^{-1} \cdot B \quad / \cdot B$  sa lijeve strane

$X^{-1}A^{-1} = X^{-1} + B$

$X^{-1}A^{-1} - X^{-1} = B$

$X^{-1}(A^{-1} - I) = B \quad / \cdot (A^{-1} - I)^{-1}$  sa desne strane

$X^{-1} = B(A^{-1} - I)^{-1} \quad /^{-1}$

$X = (A^{-1} - I) \cdot B^{-1}$

$\det A = \begin{vmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{vmatrix} \xrightarrow{I_k + II_k} \begin{vmatrix} -1 & -4 & 5 \\ -1 & -3 & 1 \\ -2 & -5 & -1 \end{vmatrix} \xrightarrow{III - I \cdot 2} \begin{vmatrix} -1 & -4 & 5 \\ -1 & -3 & 1 \\ 0 & -7 & -7 \end{vmatrix}$

$= \begin{vmatrix} -1 & -4 & 5 \\ 0 & 1 & -4 \\ 0 & 3 & -11 \end{vmatrix} = (-1) \begin{vmatrix} 1 & -4 \\ 3 & -11 \end{vmatrix} = (-1)(-11 + 12) = -1$

$A_{11} = (-1)^2 \begin{vmatrix} -3 & 1 \\ -5 & -1 \end{vmatrix} = 3 + 5 = 8$

$A_{21} = (-1)^3 \begin{vmatrix} -4 & 5 \\ -5 & -1 \end{vmatrix} = -(4 + 25) = -29$   $A_{31} = 11$

$A_{12} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -(-2 - 3) = 5$

$A_{22} = (-1)^4 \begin{vmatrix} 3 & 5 \\ 3 & -1 \end{vmatrix} = -3 - 15 = -18$   $A_{32} = 7$

$A_{13} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 3 & -5 \end{vmatrix} = -10 + 9 = -1$

$A_{23} = (-1)^5 \begin{vmatrix} 3 & -4 \\ 3 & -5 \end{vmatrix} = -(-15 + 12) = 3$   $A_{33} = -1$

$A_{kof} = \begin{bmatrix} 8 & 5 & -1 \\ -29 & -18 & 3 \\ 11 & 7 & -1 \end{bmatrix}$   $A^{-1} = (-1) \begin{bmatrix} 8 & -29 & 11 \\ 5 & -18 & 7 \\ -1 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{bmatrix}$

$\det B = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} \xrightarrow{II - I \cdot 2, III - I \cdot 2} \begin{vmatrix} 1 & 2 & 2 \\ 0 & -3 & -6 \\ 0 & -6 & -3 \end{vmatrix} = \begin{vmatrix} -3 & -6 \\ -6 & -3 \end{vmatrix} = 9 - 36 = -27$

$B^{-1} = \frac{1}{\det B} \cdot B_{kof}^T = \frac{(-1)}{-27} \begin{bmatrix} 3 & 6 & 6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix} = \frac{1}{27} \cdot 3 \cdot \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

ZA VJEŽBU (slično)

$$B^{-1} = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^{-1} - I = \begin{bmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 29 & -11 \\ -5 & 17 & -7 \\ 1 & -3 & 0 \end{bmatrix}$$

$$\begin{aligned} X &= (A^{-1} - I) \cdot B^{-1} = \begin{bmatrix} -9 & 29 & -11 \\ -5 & 17 & -7 \\ 1 & -3 & 0 \end{bmatrix} \cdot \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \\ &= \frac{1}{9} \begin{bmatrix} 27 & 33 & -87 \\ 15 & 21 & -51 \\ -5 & -1 & 8 \end{bmatrix} \end{aligned}$$

$$X = \begin{bmatrix} 3 & \frac{11}{3} & -\frac{29}{3} \\ \frac{5}{3} & \frac{7}{3} & -\frac{17}{3} \\ -\frac{5}{9} & -\frac{1}{9} & \frac{8}{9} \end{bmatrix}$$

řešené maticové  
jednice

# Riješiti matricnu jednačinu  $A \cdot X^{-1} \cdot B = B \cdot A$ , ako je  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  i  $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ .

Rj.

$$A X^{-1} B = B \cdot A \quad / \cdot A^{-1} \text{ sa lijeve strane}$$

$$X^{-1} B = A^{-1} B \cdot A \quad / \cdot B^{-1} \text{ sa desne strane}$$

$$X^{-1} = A^{-1} B \cdot A \cdot B^{-1} \quad /^{-1}$$

$$X = B A^{-1} B^{-1} A$$

$$A^{-1} = \frac{1}{\det A} \cdot A_{\text{kof}}^T$$

$$\det A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$A_{\text{kof}} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$A_{11} = 1 \quad A_{21} = -1$$

$$A_{12} = 0 \quad A_{22} = 1$$

$$A_{\text{kof}}^T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{\det B} \cdot B_{\text{kof}}^T$$

$$B_{11} = 1$$

$$B_{\text{kof}} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$B_{12} = -1$$

$$B^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\det B = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$B_{21} = 0$$

$$B_{\text{kof}}^T = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$B_{22} = 1$$

$$B \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$B^{-1} \cdot A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$X = B A^{-1} \cdot B^{-1} A =$$

$$= \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ traženo rješenje}$$

# Riješiti matricnu jednačinu:  $AX - 2B = 3X + A$  gdje je

$$A = \begin{bmatrix} 6 & 1 & 0 \\ 0 & 5 & 2 \\ 0 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}.$$

Rj:  $AX - 2B = 3X + A$

$$M = A - 3I = \begin{bmatrix} 6 & 1 & 0 \\ 0 & 5 & 2 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$AX - 3X = 2B + A$$

$$\underbrace{(A - 3I)}_M X = \underbrace{2B + A}_N$$

$$= \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$MX = N \quad / \cdot M^{-1} \text{ sa lijeve str.}$$

$$N = 2B + A = \begin{bmatrix} -2 & 4 & 0 \\ 4 & 6 & 2 \\ 8 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 6 & 1 & 0 \\ 0 & 5 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$M^{-1}MX = M^{-1}N$$

$$X = M^{-1}N$$

$$= \begin{bmatrix} 4 & 5 & 0 \\ 4 & 11 & 4 \\ 8 & 0 & 10 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det M} M_{\text{kof}}^T$$

$$\det M = \begin{vmatrix} 3 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 3 \cdot 2 \cdot 1 = 6$$

$$M_{11} = (-1)^2 \begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} = 2$$

$$M_{21} = (-1)^3 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$$

$$M_{12} = (-1)^3 \begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0$$

$$M_{22} = (-1)^4 \begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} = 3$$

$$M_{13} = (-1)^4 \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{23} = (1)^5 \begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{31} = (-1)^4 \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2$$

$$M_{32} = (-1)^5 \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = -6$$

$$M_{33} = (-1)^6 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} = 6$$

$$M_{\text{kof}} = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 0 \\ 2 & -6 & 6 \end{bmatrix},$$

$$M_{\text{kof}}^T = \begin{bmatrix} 2 & -1 & 2 \\ 0 & 3 & -6 \\ 0 & 0 & 6 \end{bmatrix}$$

$$M^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -1 & 2 \\ 0 & 3 & -6 \\ 0 & 0 & 6 \end{bmatrix},$$

$$X = M^{-1}N = \frac{1}{6} \begin{bmatrix} 2 & -1 & 2 \\ 0 & 3 & -6 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 4 & 5 & 0 \\ 4 & 11 & 4 \\ 8 & 0 & 10 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 20 & -1 & 16 \\ -36 & 33 & 48 \\ 48 & 0 & 60 \end{bmatrix}$$

$$8 - 4 + 16$$

$$0 + 12 - 48$$

$$10 - 11 + 0$$

$$0 + 33 + 0$$

$$0 - 4 + 20$$

$$12 - 60$$

$$X = \begin{bmatrix} \frac{10}{3} & -\frac{1}{6} & \frac{8}{3} \\ -6 & \frac{11}{2} & 8 \\ 8 & 0 & 10 \end{bmatrix} \text{ traženo rješenje}$$



⊕ Riješiti matricnu jednačinu  $(XA+B)^{-1}(XC+B)=C$ ,

ako je  $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ ;  $C = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

Rj.  $(XA+B)^{-1}(XC+B)=C$  /  $(XA+B)$  sa lijeve strane

$(XA+B)(XA+B)^{-1}(XC+B) = (XA+B) \cdot C$

$I$   $XC+B = XAC+BC$

$X = B(C-I)(C-AC)^{-1}$

$XC - XAC = BC - B$

$C^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$X(C-AC) = BC-B$  /  $(C-AC)^{-1}$  sa desne strane

$B(C^{-1}) = \begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 4 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$AC = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 8 \\ 0 & -2 & 2 \\ 0 & 0 & 6 \end{bmatrix}$

Označimo sa

$D = C - AC = \begin{bmatrix} -2 & 4 & -6 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

Izračunajmo  $D^{-1}$ .

$D^{-1} = \frac{1}{\det D} D_{kof}^T$

$D_{11} = (-1)^2 \begin{vmatrix} 1 & 0 \\ 0 & -4 \end{vmatrix} = -4$

$D_{21} = (-1)^3 \begin{vmatrix} 4 & -6 \\ 0 & -4 \end{vmatrix} = 16$

$D_{31} = (-1)^4 \begin{vmatrix} 4 & -6 \\ 1 & 0 \end{vmatrix} = 6$

$D_{12} = (-1)^2 \begin{vmatrix} 0 & 0 \\ 0 & -4 \end{vmatrix} = 0$

$D_{22} = (-1)^4 \begin{vmatrix} -2 & -6 \\ 0 & -4 \end{vmatrix} = 8$

$D_{32} = (-1)^5 \begin{vmatrix} -2 & -6 \\ 6 & 0 \end{vmatrix} = 0$

$D_{13} = (-1)^4 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$D_{23} = (-1)^5 \begin{vmatrix} -2 & 4 \\ 0 & 0 \end{vmatrix} = 0$

$D_{33} = (-1)^6 \begin{vmatrix} -2 & 4 \\ 0 & 1 \end{vmatrix} = -2$

$\det D = \begin{vmatrix} -2 & 4 & -6 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{vmatrix} = (-4) \begin{vmatrix} -2 & 4 \\ 0 & 1 \end{vmatrix} = 8$

$D_{kof} = \begin{bmatrix} -4 & 0 & 0 \\ 16 & 8 & 0 \\ 6 & 0 & -2 \end{bmatrix}$

$D_{kof}^T = \begin{bmatrix} -4 & 16 & 6 \\ 0 & 8 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} -\frac{1}{2} & 2 & \frac{3}{4} \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$

$X = B(C-I)(C-AC)^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & 16 & 6 \\ 0 & 8 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$X = \frac{1}{8} \begin{bmatrix} 16 & -32 & -30 \\ 0 & 16 & 2 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -\frac{15}{4} \\ 0 & 2 & \frac{1}{4} \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$  traženo rješenje

#) Riješiti matricnu jednačinu  $XAB=C$ ,  $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ ,

$B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix}$ ,  $C = [0 \ 4 \ 4]$ .

R.)  $XAB=C \quad | \cdot (AB)^{-1}$  sa desne strane

$X(AB)(AB)^{-1} = C \cdot (AB)^{-1}$

$AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 4 & 1 \\ -1 & 2 & 3 \end{bmatrix}$

$X = C \cdot (AB)^{-1}$

$\det(AB) = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 4 & 1 \\ -1 & 2 & 3 \end{vmatrix} = (-2) \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = (-2)(3+1) = -8$

AB označimo sa  $M$ , nađimo  $M^{-1}$

$M_{11} = (-1)^2 \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = 10$

$M_{21} = (-1)^3 \begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix} = -6$

$M_{31} = (-1)^4 \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix} = 2$

$M_{12} = (-1)^3 \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = -4$

$M_{22} = (-1)^4 \begin{vmatrix} 0 & 0 \\ -1 & 3 \end{vmatrix} = 0$

$M_{32} = (-1)^5 \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0$

$M_{13} = (-1)^4 \begin{vmatrix} 1 & 4 \\ -1 & 2 \end{vmatrix} = 6$

$M_{23} = (-1)^5 \begin{vmatrix} 0 & 2 \\ -1 & 2 \end{vmatrix} = -2$

$M_{33} = (-1)^6 \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix} = -2$

$M_{kof} = \begin{bmatrix} 10 & -4 & 6 \\ -6 & 0 & -2 \\ 2 & 0 & -2 \end{bmatrix}$ ,

$M_{kof}^T = \begin{bmatrix} 10 & -6 & 2 \\ -4 & 0 & 0 \\ 6 & -2 & -2 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{\det M} \cdot M_{kof}^T = \frac{-1}{8} \begin{bmatrix} 10 & -6 & 2 \\ -4 & 0 & 0 \\ 6 & -2 & -2 \end{bmatrix} = \begin{bmatrix} -5/4 & 3/4 & -1/4 \\ 1/2 & 0 & 0 \\ -3/4 & 1/4 & 1/4 \end{bmatrix}$

$X = C \cdot (AB)^{-1} = [0 \ 4 \ 4] \cdot \left(-\frac{1}{8}\right) \begin{bmatrix} 10 & -6 & 2 \\ -4 & 0 & 0 \\ 6 & -2 & -2 \end{bmatrix} = \left(-\frac{1}{8}\right) [8 \ -8 \ -8]$

$X = [-1 \ 1 \ 1]$  rješenje matricne jednačine